

QP Code **12440**

(3 Hours)

[Total Marks : 80

- N.B. : (1) Question No. 1 is **compulsory**.
 (2) Solve any **three** questions from the **remaining**.

1. (a) Find the value of μ which satisfy the equation. 5
 $A^{100} X = \mu X$, where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & -2 \\ 1 & 1 & 0 \end{bmatrix}$$

- (b) Evaluate $\int_0^{1+i} (x^2 + iy) dz$ along 5
 $y = x$ and $y = x^2$.

- (c) Find the external of the function. 5

$$\int_{x_1}^{x_2} [y^2 - y'^2 - 2y \cosh x] dx$$

- (d) Verify Cauchy-Schwartz inequality for the vectors. 5
 $u = (-4, 2, 1)$ & $v = (8, -4, -2)$

2. (a) Determine the function that gives the shortest distance between two given points. 6
 (b) Find eigen values and eigen vectors of— 6

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

- (c) Obtain Taylor's and two distinct Laurent's series expansion of $f(z) = \frac{z-1}{z^2-2z-3}$ 8
 about $z = 0$ indicating the region of convergence.

3. (a) Verify Cayley-Hamilton theorem for

6

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ hence find } A^{-2}.$$

- (b) Evaluate by using Residue theorem.

6

$$\int_0^{2\pi} \frac{d\theta}{(2 + \cos\theta)^2}$$

- (c) Solve the boundary value problem.

8

$$I = \int_0^1 \left(2xy - y^2 - y^4 \right) dx$$

given $y(0) = y(1) = 0$ by Rayleigh-Ritz method.

4. (a) Reduce the following Quadratic form

6

$$Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$$

into canonical form. Hence find its rank, index and signature.

- (b) Show that the matrix $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory.

6

- (c) (i) Show that the set $W = \{(1, x) \mid x \in \mathbb{R}\}$ is a subspace of \mathbb{R}^2 under operations $[1, x] + [1, y] = [1, x + y]$; $k[1, x] = [1, kx]$; k is any scalar.

4

- (ii) Is the set $W = \{[a, i, 1] \mid a \in \mathbb{R}\}$ a subspace of \mathbb{R}^3 under the usual addition and scalar multiplication?

4

5. (a) Find the plane curve of fixed Perimeter and maximum area.

6

- (b) Construct an orthonormal basis of \mathbb{R}^2 by applying Gram Schmidt orthogonalization to $S = \{[3, 1], [2, 2]\}$

6

- (c) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Also find diagonal form

8

and diagonalising matrix.

6. (a) Evaluate $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2 + 1)(x^2 + 4)} dx$ using Cauchy Residue Theorem. 6

(b) If $\phi(\alpha) = \oint_c \frac{ze^z}{z-\alpha} dz$ where c is $|z - 2i| = 3$ 6

find $\phi(1), \phi'(2), \phi(3), \phi'(4)$

(c) Show that the set V of positive real numbers with operations. 8

Addition : $x + y = xy$

Scalar multiplication : $kx = x^k$.

is a vector space

where x, y are any two real numbers and k is any scalar.
